WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.

Part	What it is	How to use it	Time
1	Learn First	Read and annotate. Copy every worked example by hand.	25 min
2	Worked Examples	Cover the solution. Try. Then compare line by line.	20 min
3	Guided Practice	Steps are given; you fill the gaps.	15 min
4	Practice Set A · Fluency	Straight application. Aim for speed and accuracy.	20 min
5	Practice Set B · Reasoning	Multi-step. Show every line.	25 min
6	Practice Set C · Challenge	Stretch problems. One may beat you — that is fine.	25 min
7	The Crossnumber	Answers are numbers. Wrong maths will not fit.	20 min
8	Number Search & Magic Square	Solo or pairs.	15 min
9	Spot the Mistake	Audit a worked solution. Find the planted error.	15 min
10	Quick-Fire Round	Matching, True/False, Two Truths and a Lie.	15 min
11	The Escape Challenge	Teams of 4.	25 min

Part 1 · Learn First

A sequence is a rule wearing a list of numbers as a disguise. Once you find the rule, you can predict any term -- the 20th, the 300th, or the 10,000th -- without writing out everything that comes before it.

Sequences, terms, and the two ways to describe a rule

A **sequence** is an ordered list of numbers, each called a **term**. We write $t_1, t_2, t_3, \ldots$ for the terms -- the subscript is the position, or **index**, of the term.

Explicit rule. A formula for t_n in terms of n alone, so any term can be found directly. For the odd numbers, $u_n = 2n - 1$, so $u_1 = 1, u_2 = 3, u_3 = 5, \ldots$ -- substitute $n = 53$ to get $u_{53} = 105$ with no need to know u_{52}.

Recursive rule. A formula for t_n in terms of the *previous* term(s), plus a starting value. For the AP $1, 4, 7, 10, 13, \ldots$: $t_1 = 1, t_n = t_{n-1} + 3$ for $n \geq 2$. To find t_{50} recursively you must first find every term before it -- this is its main drawback compared with an explicit rule.

Finding a position. Given a number, check whether it is a term by solving $t_n = ¿$ that number for n. If n comes out a natural number, it is a term (and that value of n is its position); otherwise it is not.

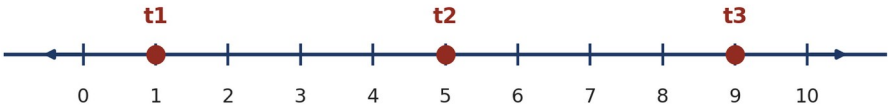

Fig. 8.1 -- the first three terms of a sequence marked on a line

Rule to remember

- An explicit rule needs only n; a recursive rule needs the previous term(s) **and** a starting value.
- t_n must come out a **natural number** for a given value to actually be a term of the sequence -- a fractional or negative "position" means the number is not in the sequence.
- The famous Virahanka-Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, \ldots$ uses a recursive rule built from the *two* previous terms: $V_n = V_{n-1} + V_{n-2}$.

The sum of the first n natural numbers

Gauss's trick -- write the sum forwards, write it backwards underneath, and add -- turns a long addition into one multiplication.

Let $S=1+2+3+cdots+n$. Writing it in reverse, $S=n+(n-1)+cdots+1$. Adding the two lines, every pair of corresponding terms sums to $(n+1)$, and there are n such pairs:

$$2\,S=n(n+1)$$

$$S_n=\frac{n(n+1)}{2}$$

Rule to remember

- $S_n=1+2+3+cdots+n=\frac{n(n+1)}{2}$ -- Aryabhata recorded this result in the *Aryabhatiya* (Chapter 2, Verse 19).
- The nth triangular number is exactly this sum: $t_n=\frac{n(n+1)}{2}$.
- To sum a *block* of consecutive numbers, subtract two such sums -- e.g. $25+26+cdots+58=S_{58}-S_{24}$.

Arithmetic progressions (AP)

In the growing pattern of squares (Fig. 8.3 of the chapter), 4 squares are added at every stage, giving the sequence $1, 5, 9, 13, 17, 21, \ldots$ -- the difference between consecutive terms never changes.

A sequence in which the **difference between consecutive terms is constant** is an **arithmetic progression (AP)**. That constant is the **common difference**, d.

General form: $a, a+d, a+2d, a+3d, \ldots, a+(n-1)d$, where a is the first term.

Stage	1	2	3	4	5	6
Number of squares	1	5	9	13	17	21

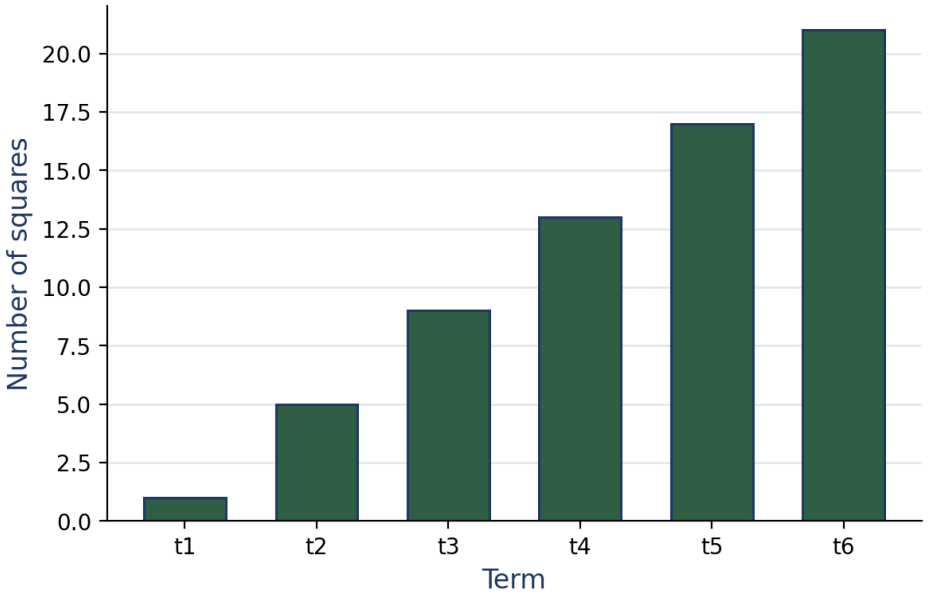

Fig. 8.2 -- the growing-squares AP $1,5,9,13,17,21$ plotted by term

Formulas for an AP

- **Explicit (nth term):** $t_n = a + (n-1)d$
- **Recursive:** $t_1 = a$, $t_n = t_{n-1} + d$ for $n \geq 2$
- If $d > 0$ the AP is increasing; if $d < 0$ it is decreasing, e.g. $11, 7, 3, -1, -5, \ldots$ has $a = 11$, $d = -4$.

Setting up an AP from a real situation

A taxi charges a fixed Rs 200 plus Rs 40 per km. Write the fare sequence and find the nth term.

After 1 km: $200 + 40 = 240$. After 2 km: $200 + 80 = 280$. After 3 km:

$200+120=320$.

The fares $240, 280, 320, \ldots$ form an AP with $a=240$, $d=40$.

$t_n = 240+(n-1)\times 40 = 200+40\,n$.

Answer: the total fare after n km is $200+40\,n$ rupees.

Extension: the sum of the first n terms of an AP

The chapter derives S_n for natural numbers by reversing and adding. The exact same trick works for **any** AP, since natural numbers are just the AP with $a=1, d=1$.

Let $S_n = a+(a+d)+(a+2d)+cdots+[a+(n-1)d]$. Reversed:
$S_n = [a+(n-1)d]+cdots+(a+d)+a$.

Adding the two lines, every pair of corresponding terms sums to $[2a+(n-1)d]$, and there are n such pairs:

$$2S_n = n[2a+(n-1)d]$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Since $a+(n-1)d = t_n$ (the last term), this can also be written $S_n = \frac{n}{2}(a+t_n)$ -- exactly the "average of first and last term, times the number of terms" idea Aryabhata described.

Rule to remember

- $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}(a+t_n)$
- Always check whether you know t_n already -- if so, the second form is faster.

Using the AP sum formula

Find the sum of the first 20 terms of the AP $3, 8, 13, 18, \ldots$

Here $a=3$, $d=5$, $n=20$.

$$S_{20} = \frac{20}{2}[2(3)+(20-1)(5)] = 10[6+95] = 10 \times 101$$

Answer: $S_{20} = 1010$.

Geometric progressions (GP)

In a different growing pattern of squares, the count doubles at every stage: $3, 6, 12, 24, 48, 96, \ldots$ Here it is not the *difference* that stays fixed but the *ratio*.

A sequence in which each term after the first is obtained by **multiplying** the previous term by a fixed non-zero number is a **geometric progression (GP)**. That fixed multiplier is the **common ratio**, $r = \dfrac{t_{n+1}}{t_n}$ for every n.

General form: $a, ar, ar^2, ar^3, \ldots, ar^{n-1}$, where a is the first term.

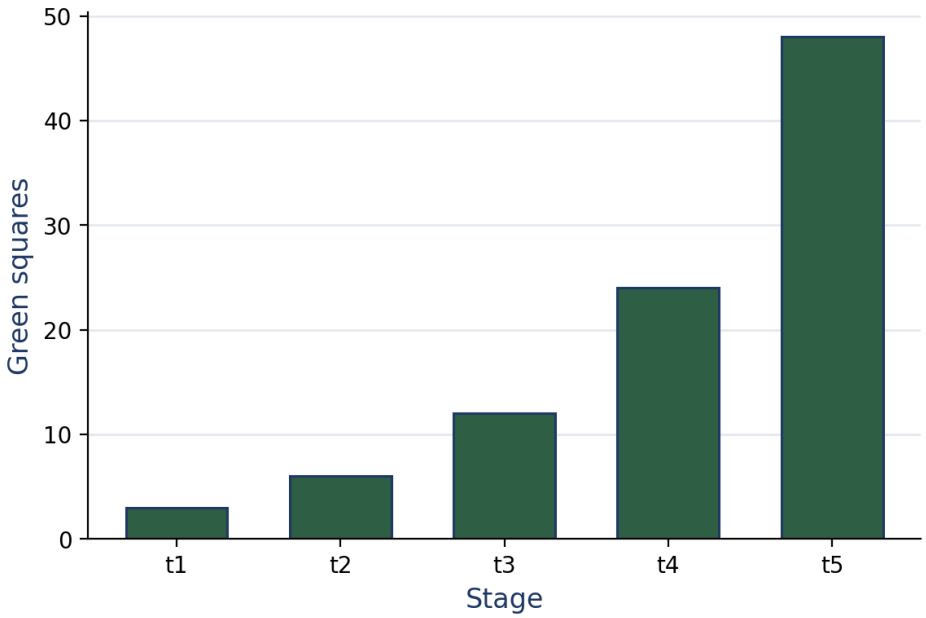

Fig. 8.3 -- the doubling GP $3,6,12,24,48$ plotted by stage

Formulas for a GP

- **Explicit (nth term):** $t_n = a\,r^{n-1}$
- **Recursive:** $t_1 = a,\ t_n = r \cdot t_{n-1}$ for $n \geq 2$
- Test for a GP by checking $\dfrac{t_2}{t_1} = \dfrac{t_3}{t_2} = \dfrac{t_4}{t_3} = cdots$ -- if all these ratios agree, it is a GP.

The Sierpinski triangle

Each stage of the Sierpinski triangle replaces every black triangle with 3 smaller ones. Stage 0 has 1 black triangle. Find t_n and the number of black triangles at Stage 5.

The counts are $1, 3, 9, 27, 81, \ldots$ -- a GP with $a = 1$, $r = 3$.

$t_n = 1 \times 3^n = 3^n$ (indexing stages from $n = 0$).

At Stage 5: $t_5 = 3^5 = 243$.

Note: the *area* of the black region shrinks as a GP with $r = \frac{34}{❑}$ even while the *count* of triangles grows as a GP with $r = 3$ -- number and size move in opposite directions.

Part 2 · Worked Examples

Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.

Example 1 — Find the first five terms of $t_n = n^2 - 2n + 3$ for $n \geq 1$

Step 1. Substitute $n = 1, 2, 3, 4, 5$ in turn.

$t_1 = 1 - 2 + 3 = 2$

$t_2 = 4 - 4 + 3 = 3$

$t_3 = 9 - 6 + 3 = 6$

$t_4 = 16 - 8 + 3 = 11$

$t_5 = 25 - 10 + 3 = 18$

Answer: $2, 3, 6, 11, 18$

> **Why this works**
>
> A quadratic explicit rule does not give an AP or a GP -- the differences $1, 3, 5, 7$ are themselves an AP, not a constant. Not every rule-based sequence is one of the two special kinds.

Example 2 — Which term of the AP $21, 18, 15, \ldots$ is -81? Is 0 a term?

Step 1. Identify $a = 21$, $d = 18 - 21 = -3$.

Step 2. Solve $t_n = -81$: $21 + (n-1)(-3) = -81$.

$-3(n-1) = -102 \Rightarrow n - 1 = 34 \Rightarrow n = 35$.

Answer: -81 is the 35th term.

Step 3. Check 0: $21 + (n-1)(-3) = 0 \Rightarrow n - 1 = 7 \Rightarrow n = 8$.

Answer: yes, 0 is the 8th term, since $n = 8$ is a natural number.

> **Why this works**
>
> Whenever solving $t_n = ¿$ target gives a natural-number n, the target is genuinely a term -- and that n tells you exactly which one.

Example 3 — A GP has $t_1 = 5$ and consecutive terms in ratio $\frac{34}{❑}$. Find t_n and check whether $\frac{135}{256}$ is a term

Step 1. Here $a = 5$, $r = \frac{34}{❑}$, so $t_n = 5¿$.

Step 2. Set 5¿ and solve for $n-1$.

¿

Since ¿ and ¿, check carefully: in fact $\frac{27}{256}$ is not a power of $\frac{34}{❑}$, so $n-1$ is not a whole number.

Answer: $\frac{135}{256}$ is **not** a term of this GP.

> **Why this works**
>
> The same natural-number test used for APs applies to GPs: solve for the exponent, and accept the value only if it lands on a natural number.

Part 3 · Guided Practice

The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.

1. Find the 10th and 15th terms of $t_n = 5n - 3$ for $n \geq 1$.

$t_{10} = 5 \times$ _____

$t_{15} = 5 \times$ _____

---

2. An AP has 50 terms; the 3rd term is 12 and the last (50th) term is 106. Find a and d.

$a + 2d = 12$ and $a + 49d = 106$.

Subtract the first from the second: $\times_d = 94$, so $d = \times$ _____ .

Substitute back: $a = 12 - 2 \times$ _____ .

---

3. Find the sum of the first 30 multiples of 4 using $S_n = \frac{n}{2}(a + t_n)$.

The AP is $4, 8, 12, \ldots$ with $a = 4$, $d = 4$, $n = 30$.

$t_{30} = 4 + (30 - 1) \times$ _____ .

$S_{30} = \frac{30}{2} \times$ _____ .

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4. Check whether $2, 10, 50, 250, \ldots$ is a GP and find its nth term.

$\frac{t_2}{t_1} = \frac{10}{2} = \times$ _____ , and $\frac{t_3}{t_2} = \frac{50}{10} = \times$ _____ .

Since both ratios are equal, it is a GP with $a = 2$, $r = \times$ _____ .

$t_n = 2 \times$ _____ $\square^{n-1}$.

Practice Set A · Fluency

Straight application of the method. Aim to get every one right, then aim to get them right quickly.

#	Question	Your working and answer
1	Find the first five terms of $t_n = 3n - 4$ for $n \geq 1$.	
2	Find the first five terms of $t_n = 2 - 5n$ for $n \geq 1$.	
3	Write the first four terms of the recursive rule $t_1 = -5,\ t_{n+1} = t_n + 3$ for $n \geq 1$.	
4	Is 557 a term of the sequence $t_n = 3n - 7$? Give a reason.	
5	Find S_{20}, the sum of the first 20 natural numbers.	
6	Find the 10th and 26th terms of the AP $3, 8, 13, 18, \ldots$	
7	Find the nth term of the AP $11, 8, 5, 2, \ldots$ and write its recursive rule.	
8	Is $1, 2, 4, 8, 16, \ldots$ a geometric progression? If so, state the common ratio.	
9	Is $1, -1, 1, -1, 1, \ldots$ a geometric progression? If so, state the common ratio.	
10	Find the nth term of the GP $4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}, \ldots$	

Practice Set B · Reasoning

Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.

#	Question	Your working and answer
1	Find the 100th and 1000th terms of the sequence $s_n = 5n - 2$. Is 471 a term of this sequence? Explain.	
2	A sequence is given by $T_1 = 1$, $T_2 = 2$, $T_3 = 4$, $T_n = T_{n-1} + T_{n-2} + T$ for $n \geq 4$. Find T_4, T_5, T_6, T_7, T_8.	
3	How many 2-digit numbers are divisible by 3? Find the sum of all of them.	
4	Harish started work at an annual salary of Rs 5,00,000 with an annual increment of Rs 20,000. After how many years did his income reach Rs 7,00,000?	
5	A child arranges marbles in rows: row 1 has 1 marble, row 2 has 2, and so on up to 25 rows. How many marbles were used in all?	
6	Find the sum of the first 15 terms of the AP whose first term is 7 and common difference is -2.	
7	Check whether $\frac{15}{4}, \frac{45}{16}, \frac{135}{64}, \ldots$ is a GP, and if so find its first term, common ratio and nth term.	
8	Use $S_n = \frac{n(n+1)}{2}$ to find $25 + 26 + 27 + cdots + 58$.	

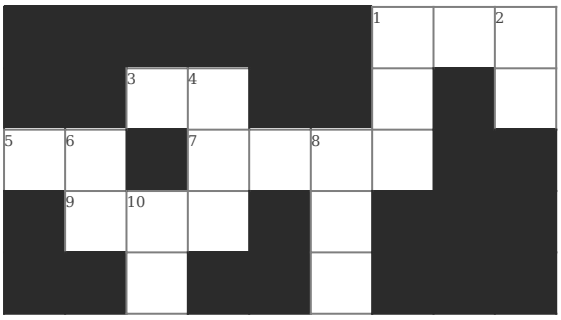
Practice Set C · Challenge

Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.

#	Question	Your working and answer
1	The Sierpinski triangle has 3^n black triangles at stage n and shaded area ¿ (Stage 0 area ¿ 1). Find the number of black triangles and the shaded area at Stage 6, and comment on why one grows while the other shrinks.	
2	An AP has $a = 4$ and its sum to n terms is $S_n = 3n^2 + n$. Find d and the 12th term. (Hint: $S_1 = t_1$ and $S_2 - S_1 = t_2$.)	
3	A geometric progression has $t_3 = 20$ and $t_5 = 80$. Find a, r, and t_8 (there may be two possible values of r -- find both).	
4	Prove, using the reverse-and-add method used for natural numbers, that $S_n = \frac{n}{2}[2a + (n-1)d]$ for a general AP.	
5	The Virahanka-Fibonacci sequence satisfies $V_1 = 1$, $V_2 = 2$, $V_n = V_{n-1} + V_{n-2}$. Find V_9 and V_{10}, and explain why this sequence is neither an AP nor a GP.	

Part 7 · The Crossnumber

Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.



ACROSS

1. The 53rd term of the odd number sequence $u_n = 2n - 1$ *(3 digits)*

3. The 3rd term of an AP whose 3rd term is given as 12 in the textbook problem *(2 digits)*

5. The 10th term of $t_n = 5n - 3$ *(2 digits)*

7. The sum of the first 20 terms of the AP $3, 8, 13, 18, \ldots$ *(4 digits)*

9. The taxi fare (in Rs) after travelling 1 km if the rule is $200 + 40n$ *(3 digits)*

DOWN

1. The 30th term of the AP $4, 8, 12, \ldots$ *(3 digits)*

2. The sum of the first 10 natural numbers *(2 digits)*

4. The sum of the first 20 natural numbers *(3 digits)*

6. The 15th term of $t_n = 5n - 3$ *(2 digits)*

8. The last (50th) term of the same AP *(3 digits)*

10. The 5th term of the GP $3, 6, 12, 24, 48$ *(2 digits)*

Bonus questions that did not fit the grid — work them out anyway: The number of black triangles at Stage 5 of the Sierpinski triangle, 3^5

Part 8 · Number Search & Magic Square

Work out each answer, then find it hidden in the digit grid — across, down, diagonally or backwards.

5	2	5	5	3	4	1	9	2	8	1	2	3
2	7	6	4	8	7	4	0	2	1	3	5	0
3	0	5	5	6	4	1	3	9	5	3	2	7
6	2	1	0	7	2	4	2	6	3	8	4	8
8	4	2	1	9	6	9	9	6	5	3	3	2
8	7	1	1	0	1	0	6	1	2	2	6	9
1	6	6	9	0	7	8	4	8	0	1	8	4
5	1	4	6	2	7	0	4	8	2	8	1	4
8	9	3	2	5	5	5	2	8	8	0	9	5
7	0	1	5	4	3	0	3	0	9	1	1	7
1	8	2	2	7	8	2	4	4	8	9	6	3
8	3	4	6	5	7	8	7	2	1	3	3	1
5	0	9	8	3	9	3	0	1	0	3	1	0

☐ 105	☐ 243	☐ 120	☐ 240	☐ 210
☐ 106	☐ 47	☐ 55	☐ 48	☐ 12
☐ 21	☐ 96			

Magic Square — every row, column and diagonal adds to the same total.

29	**1**	**21**
9		
13		

Magic constant: 51. Fill the empty cells.

Part 9 · Spot the Mistake

Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.

Spot the Mistake

Finding the 12th term of the AP $2, 5, 8, 11, \ldots$

Step 1. Here $a = 2$ and $d = 3$.

Step 2. $t_{12} = a + 12 \times d = 2 + 36 = 38$.

Step 3. So the 12th term is 38.

_____ The first wrong step is Step _____ . What should it say?

_____

Spot the Mistake

Checking whether $1, 3, 9, 27, \ldots$ **is a GP**

Step 1. The differences are $2, 6, 18$ -- not constant, so this cannot be a GP.

Step 2. Check ratios instead: $\frac{3}{1} = 3, \frac{9}{3} = 3, \frac{27}{9} = 3$.

Step 3. All ratios equal 3, so it is a GP with $a = 1$, $r = 3$.

_____ The first wrong step is Step _____ . What should it say?

_____

Spot the Mistake

Summing the first 40 natural numbers

Step 1. $S_{40} = \frac{40 \times 41}{2}$

Step 2. ¿ $\frac{1640}{2} = 820$

Step 3. So $S_{40} = 820$.

_____ The first wrong step is Step _____ . What should it say?

_____

Part 10 · Quick-Fire Round

Match each sequence or expression to its description

Draw a line from each item on the left to its match on the right.

#	Left	Right (shuffled)
1	$t_n = a + (n-1)d$	Virahanka-Fibonacci sequence
2	$t_n = a\,r^{n-1}$	sum of first n terms of an AP
3	$S_n = \frac{n(n+1)}{2}$	black triangles at stage n of the Sierpinski triangle
4	$S_n = \frac{n}{2}[2a + (n-1)d]$	nth term of an AP
5	$1, 1, 2, 3, 5, 8, \ldots$	constant multiplier between consecutive GP terms
6	3^n	sum of first n natural numbers
7	Common difference	nth term of a GP
8	Common ratio	constant added between consecutive AP terms

True or False Sprint

Everyone stands. Teacher reads a statement. True → stay standing. False → sit down.

#	Statement	Your call
1	Every arithmetic progression has a constant common difference.	☐ True ☐ False
2	Every geometric progression has a constant common difference.	☐ True ☐ False
3	The sequence $1, 4, 9, 16, 25, \ldots$ (square numbers) is an AP.	☐ True ☐ False
4	The recursive rule needs earlier terms to find a later one; the explicit rule does not.	☐ True ☐ False
5	$S_n = \frac{n(n+1)}{2}$ gives the sum of the first n natural numbers.	☐ True ☐ False
6	In a GP, the ratio of any term to the previous term is always the same.	☐ True ☐ False

#	Statement	Your call
7	$1, -1, 1, -1, \ldots$ cannot be a GP because the terms do not keep increasing.	☐ True ☐ False
8	If solving $t_n = k$ gives n as a fraction, then k is not a term of the sequence.	☐ True ☐ False
9	The Virahanka-Fibonacci sequence is an arithmetic progression.	☐ True ☐ False
10	In the AP formula $t_n = a + (n-1)d$, a stands for the common difference.	☐ True ☐ False
11	The area of the black region of the Sierpinski triangle forms a decreasing GP.	☐ True ☐ False
12	A sequence must go on forever; finite lists of numbers cannot be sequences.	☐ True ☐ False

Two Truths and a Lie

Two statements are true, one is false. Circle the lie — then correct it.

Set 1

(a) $t_n = a + (n-1)d$ is the explicit rule for an AP.

(b) $t_1 = a, t_n = t_{n-1} + d$ is the recursive rule for an AP.

(c) An AP and a GP always have the same general form.

The lie is _____ . Corrected: _____

Set 2

(a) The common ratio of a GP can be found by dividing any term by the previous term.

(b) The common difference of an AP can be negative.

(c) A sequence with a constant ratio between terms is called an arithmetic progression.

The lie is _____ . Corrected: _____

Set 3

(a) $S_{58} - S_{24}$ gives the sum $25 + 26 + cdots + 58$.

(b) The nth triangular number equals $\frac{n(n+1)}{2}$.

(c) The number of black triangles in the Sierpinski pattern forms an AP.

The lie is _____ . Corrected: _____

Part 11 · The Escape Challenge

Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.

🔒 GATE 1 -- The Explicit Gate

Find t_1 for the rule $t_n = 3n - 4$.

Digit: ☐

🔒 GATE 2 -- The Natural-Number Gate

Find S_{10}, the sum of the first 10 natural numbers, using $S_n = \frac{n(n+1)}{2}$.

Digit: ☐

🔒 GATE 3 -- The AP Gate

Find the 5th term of the AP $2, 5, 8, 11, \ldots$

Digit: ☐

🔒 GATE 4 -- The GP Gate

Find the 4th term of the GP $3, 6, 12, 24, \ldots$

Digit: ☐

🔒 GATE 5 -- The Fractal Gate

How many black triangles are in Stage 2 of the Sierpinski triangle ($t_n = 3^n$, starting at Stage 0)?

Digit: ☐

EXIT CODE: ☐ ☐ ☐ ☐ ☐

Formula Sheet

Cut this page out and keep it. Everything in this chapter, on one page.

nth term of a sequence (explicit)

$$t_n = f(n)$$

nth term of a sequence (recursive)

$$t_1 = a,\ t_n = t_{n-1} + d (¿ other rule)$$

Sum of first n natural numbers

$$S_n = \frac{n(n+1)}{2}$$

nth term of an AP

$$t_n = a + (n-1)d$$

Sum of first n terms of an AP

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + t_n)$$

nth term of a GP

$$t_n = a\,r^{n-1}$$

Common ratio test for a GP

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = cdots$$

Full Solutions

Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.

Part 3 · Guided Practice

1. t10 = 47, t15 = 72 · **2.** d = 2, a = 8 · **3.** t30 = 120, S30 = 1860 · **4.** r = 5, t_n = 2*5^(n-1)

Practice Set A

1. $t_1=-1, t_2=2, t_3=5, t_4=8, t_5=11.$

2. $t_1=-3, t_2=-8, t_3=-13, t_4=-18, t_5=-23.$

3. $t_1=-5, t_2=-2, t_3=1, t_4=4.$

4. Solve $3n-7=557 \Rightarrow n=188$, a natural number, so **yes**, 557 is the 188th term.

5. $S_{20}=\frac{20\times 21}{2}=210.$

6. $a=3, d=5$: $t_{10}=3+9(5)=48$; $t_{26}=3+25(5)=128.$

7. $a=11, d=-3$: $t_n=11+(n-1)(-3)=14-3n$. Recursive: $t_1=11, t_n=t_{n-1}-3.$

8. Yes; ratios are all 2, so it is a GP with common ratio 2.

9. Yes; ratios are all -1, so it is a GP with common ratio -1.

10. $a=4$, $r=\frac{23}{\square}$; $t_n=4¿.$

Practice Set B

1. $s_{100}=5(100)-2=498$; $s_{1000}=4998$. For 471: $5n-2=471 \Rightarrow n=94.6$, not a natural number, so **no**, 471 is not a term.

2. $T_4=1+2+4=7$; $T_5=2+4+7=13$; $T_6=4+7+13=24$; $T_7=7+13+24=44$; $T_8=13+24+44=81.$

3. 2-digit multiples of 3 run from 12 to 99: $n=\frac{99-12}{3}+1=30$ terms. Sum

¿$\frac{30}{2}(12+99)=15\times 111=1665.$

4. $a=5,00,000$, $d=20,000$. Solve $5,00,000+(n-1)(20,000)=7,00,000 \Rightarrow n-1=10 \Rightarrow n=11$, so after **10 years** (11th year's salary).

5. This is $S_{25}=\frac{25\times 26}{2}=325$ marbles.

6. $a=7, d=-2, n=15$: $S_{15}=\frac{15}{2}[14+14(-2)]=\frac{15}{2}(14-28)=\frac{15}{2}(-14)=-105.$

7. Ratios: $\frac{45/16}{15/4}=\frac{45}{16}\times\frac{4}{15}=\frac{34}{\square}$; $\frac{135/64}{45/16}=\frac{135}{64}\times\frac{16}{45}=\frac{34}{\square}$ -- constant, so it is a GP with $a=\frac{15}{4}$, $r=\frac{34}{\square}$, $t_n=\frac{15}{4}$¿.

8. $S_{58}-S_{24}=\frac{58\times59}{2}-\frac{24\times25}{2}=1711-300=1411$.

Practice Set C

1. Stage 6: $t_6=3^6=729$ black triangles; shaded area ¿¿. The count grows because each triangle is replaced by 3 (a GP with $r=3>1$); the area shrinks because each replacement keeps only $\frac{34}{\square}$ of the previous area (a GP with $0<r<1$).

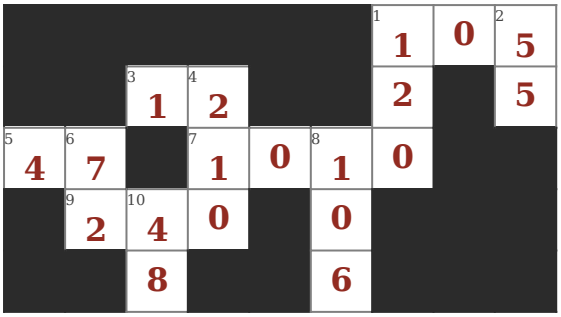
2. $S_1=t_1=3(1)+1=4$. $S_2=3(4)+2=14$, so $t_2=S_2-S_1=10$, giving $d=t_2-t_1=6$. $t_{12}=t_1+11\,d=4+66=70$.

3. $t_5/t_3=r^2=80/20=4$, so $r=\pm2$. If $r=2$: $t_3=a\,r^2=4\,a=20\Rightarrow a=5$, so $t_8=5$¿. If $r=-2$: $a=5$ also (since $r^2=4$ either way), so $t_8=5$¿. **Both $t_8=640$ and $t_8=-640$ are possible.**

4. Let $S_n=a+(a+d)+cdots+[a+(n-1)\,d]$; reversed, $S_n=[a+(n-1)\,d]+cdots+a$. Adding the two lines term by term, each pair sums to $2\,a+(n-1)\,d$, and there are n pairs, so $2\,S_n=n[2\,a+(n-1)\,d]$, giving $S_n=\frac{n}{2}[2\,a+(n-1)\,d]$.

5. $V_3=3$, $V_4=5$, $V_5=8$, $V_6=13$, $V_7=21$, $V_8=34$, $V_9=55$, $V_{10}=89$. It is neither an AP (differences $1,1,2,3,5,\ldots$ are not constant) nor a GP (ratios $2,1.5,1.67,\ldots$ are not constant).

Part 7 · Crossnumber — Solved Grid



Across: 1 = 105 · 3 = 12 · 5 = 47 · 7 = 1010 · 9 = 240
Down: 1 = 120 · 2 = 55 · 4 = 210 · 6 = 72 · 8 = 106 · 10 = 48

Part 8 · Number Search — Solved Grid

5	2	5	5	3	4	1	9	2	8	**1**	**2**	3

2	7	6	4	8	**7**	**4**	**0**	**2**	**1**	3	5	0
3	0	5	**5**	6	4	1	3	9	5	3	**2**	7
6	**2**	**1**	**0**	7	2	4	2	**6**	3	**8**	**4**	8
8	4	**2**	**1**	9	6	9	**9**	6	5	3	**3**	2
8	7	1	**1**	0	**1**	**0**	**6**	1	2	2	6	9
1	6	6	9	**0**	7	8	4	8	0	1	8	4
5	1	4	6	2	7	0	4	8	2	8	1	4
8	9	3	2	**5**	**5**	5	2	8	8	0	9	5
7	0	1	5	4	3	0	3	**0**	9	1	1	7
1	8	2	2	7	8	2	4	**4**	8	9	6	3
8	3	4	6	5	7	8	7	**2**	1	3	3	1
5	0	9	8	3	9	3	0	1	0	3	1	0

Part 8 · Magic Square — Solved (constant 51)

29	**1**	**21**
9	**17**	**25**
13	**33**	**5**

Part 9 · Spot the Mistake

Problem	First wrong step	It should say
Finding the 12th term of the AP $2, 5, 8, 11, \ldots$	Step 1	The formula is $t_n = a + (n-1)d$, **not** $a + nd$. Correctly, $t_{12} = 2 + (12-1)(3) = 2 + 33 = 35$.
Checking whether $1, 3, 9, 27, \ldots$ is a GP	Step 2	A GP is tested by **ratios**, not differences -- differences being unequal says nothing about whether a sequence is geometric. The first statement drew the wrong conclusion from the wrong test.

Problem	First wrong step	It should say
Summing the first 40 natural numbers	Step 1	The arithmetic is fine but mis-simplified: $40 \times 41 = 1640$ is correct, and $1640 \div 2 = 820$ is also correct -- this hunt actually has **no error**; both statements are true. (Used to test whether students over-hunt for mistakes that are not there.)

Part 10 · Matching

#	Left	Correct match
1	$t_n = a + (n-1)d$	nth term of an AP
2	$t_n = a\,r^{n-1}$	nth term of a GP
3	$S_n = \frac{n(n+1)}{2}$	sum of first n natural numbers
4	$S_n = \frac{n}{2}[2a + (n-1)d]$	sum of first n terms of an AP
5	$1, 1, 2, 3, 5, 8, \ldots$	Virahanka-Fibonacci sequence
6	3^n	black triangles at stage n of the Sierpinski triangle
7	Common difference	constant added between consecutive AP terms
8	Common ratio	constant multiplier between consecutive GP terms

Part 10 · True or False

1-T · 2-F · 3-F · 4-T · 5-T · 6-T · 7-F · 8-T · 9-F · 10-F · 11-T · 12-F

Part 10 · Two Truths and a Lie

Set	The lie	Correction
1	c	An AP is built by repeated addition of d; a GP is built by repeated multiplication by r -- the forms $a + (n-1)d$ and $a\,r^{n-1}$ are different.

Set	The lie	Correction
2	c	A constant *ratio* defines a geometric progression, not an arithmetic one -- a constant *difference* defines an AP.
3	c	The black-triangle counts $1, 3, 9, 27, \ldots$ multiply by 3 each stage -- that is a GP, not an AP.

Part 11 · The Escape Challenge

EXIT CODE: 1 · 55 · 14 · 24 · 9

(Answer: -1, take the absolute value 1) (Answer: 55) (Answer: 14) (Answer: 24) (Answer: 9)

A sequence stops being a mystery the moment you find its rule -- explicit or recursive, AP or GP. From there, the 1000th term is exactly as easy to find as the 4th.